KP Equation for Electrostatic Solitary Waves in a Plasma containing Nonextensive Electrons

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Motivation:

The study of ion-acoustic solitary waves in multicomponent plasmas has an important place in both theoretical and experimental areas of nonlinear plasma processes. Over the last many years, the generalization of Boltzmann-Gibbs-Shannon (BGS) entropy has been used in a wide range of phenomenon characterized by nonextensitivity (q-nonextenisve distribution). The q-nonextensive distribution is more general than nonthermal distribution to study the dynamics of solitary waves in most of the space and astrophysical plasma environments. The reductive perturbation method is employed to derive the KP equation to investigate the small amplitude solitary structures.

Fluid Model Equations

Continuity Eq.

$$\frac{\partial n}{\partial t}+\frac{n\partial u}{\partial x}+\frac{u\partial n}{\partial x}+\frac{n\partial v}{\partial y}+\frac{v\partial n}{\partial y}=0$$

Momentum Eq.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial \varphi}{\partial x}+\frac{\sigma}{n}\frac{\partial p}{\partial x}=0$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+u\frac{\partial v}{\partial y}+\frac{\partial \varphi}{\partial y}+\frac{\sigma}{n}\frac{\partial p}{\partial y}=0$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+2p\frac{\partial u}{\partial x}+2p\frac{\partial v}{\partial y}=0$$

Poisson's Equation

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=-n+n_e$$

Electron density

$$\phi=\left(1+(q+1)\varphi\right)^{(q+1)/2(q-1)}$$

Scaling parameters:

$$u = \frac{u_i}{v_0},\ n = \frac{n_i}{n_{i0}},\ \phi = \frac{\phi}{\phi_0},\ \phi_0 = \frac{K_B T_e}{e},$$

$$p = \frac{p}{p_0},\ p_0 = n_0 K_B T_e,\ \sigma = \frac{T_i}{T_e}$$

$$\lambda_{De} = \left(\frac{K_B T_e}{4\pi e^2 n_0}\right)^{1/2},\quad \omega_{p_i} = \left(\frac{4\pi\ n_{i0} e^2}{m_i}\right)^{1/2},\quad C_s = \sqrt{\frac{K_B T_e}{m_i}}$$

Reductive Perturbation Method

Stretching co-ordinates:

$$\xi = \varepsilon(x - \lambda_0 t),\quad \tau = \varepsilon^3 t,\quad \eta = \varepsilon^2 y$$

Expanded variables:

$$A = A_0 \sum_{n=1}^{\infty} \varepsilon^{2n} A_n$$

$$v = \varepsilon^3 v_1 + \varepsilon^5 v_2 + \ldots\ldots\ldots$$

$$A(n, p, u, \phi); \qquad n_0 = 1, p_0 = 1, u_0 = 0, \phi_0 = 0$$

Dispersion relation:

$$\lambda_0 = \sqrt{2\sigma + \frac{1}{c_1}}; c_1 = \frac{q+1}{2}$$

Elimination of second order quantities yields

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \varphi}{\partial \tau}+\alpha \varphi \frac{\partial \varphi}{\partial \xi}+\beta \frac{\partial^3 \varphi}{\partial \xi^3}\right]+\delta \frac{\partial^2 \varphi}{\partial \eta^2}=0$$

KP Equation

$$\alpha=\frac{\lambda_0((q+1)^3 \lambda_0{}^2+2(q+1)(q-3)+4\sigma(q+1)^3+4(q+1)^2)}{4(q+1)+2(q+1)^2 \lambda_0{}^2+4\sigma(q+1)^2}$$

$$\beta=\frac{4\lambda_0}{2(q+1)+(q+1)^2 \lambda_0+2\sigma(q+1)^2}$$

$$\delta=\frac{2(2+\sigma(q+1)^2)}{\lambda_0(2(q+1)+(q+1)^2 \lambda_0{}^2+2\sigma(q+1)^2}$$

Solution of the KP equation

Using transformation $X = \xi + \eta - V\tau$

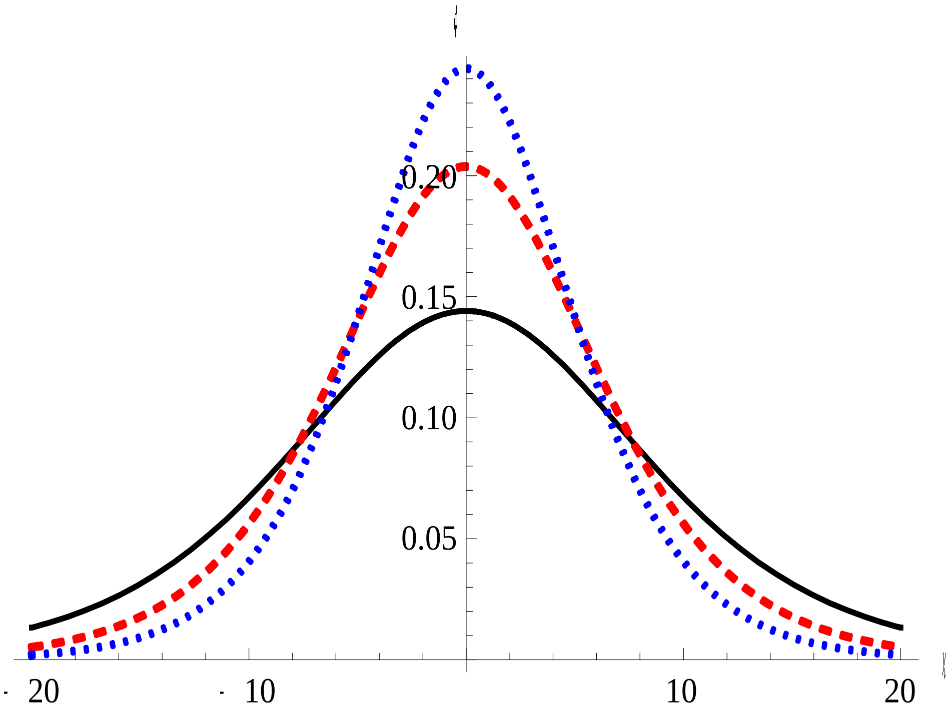
$$\phi(X) = \phi_m \, Sech^2 (X / L)$$

Amplitude: $$\phi_m = \frac{3(V - \delta)}{\alpha}$$

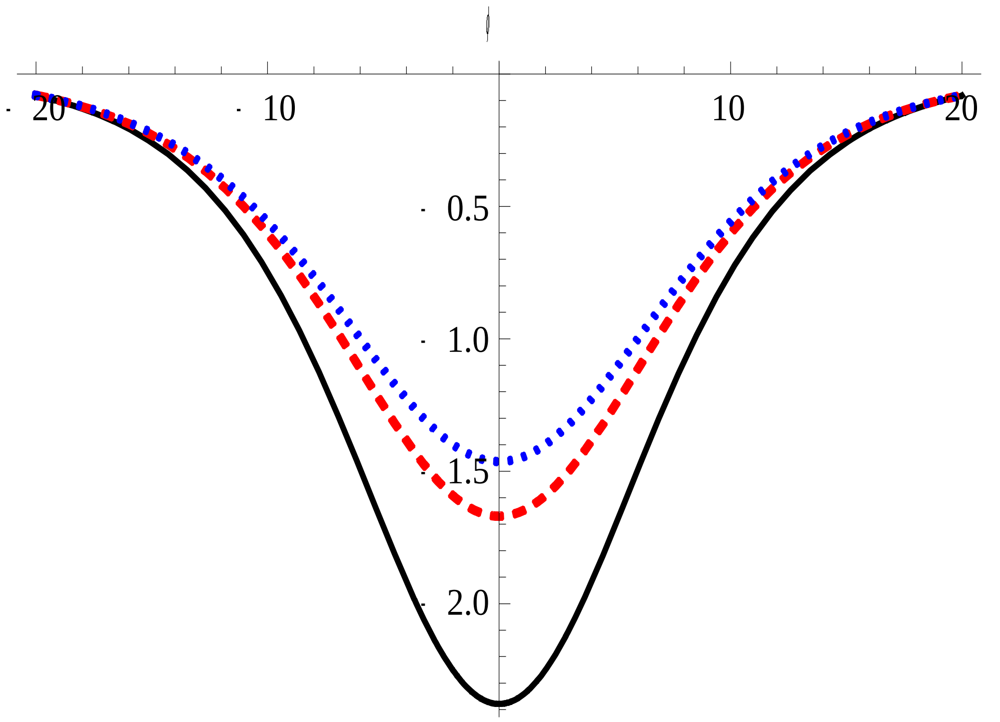
Width: $$L = \sqrt{\frac{4\beta}{V - \delta}}$$

Parametric Analysis

Effect of Nonextensitivity on Pulse profile of IASWs



Black: q=0.5, Red: q=0.6, Blue: q=0.7



Black: q=-0.1, Red: q=-0.125, Blue: q=-0.135

Conclusions

► Both positive and negative potential solitary structures are observed

► Nonextensitivity influences the characteristics of solitary structures, i.e., amplitude (width) increases (decreases).

► Amplitude (width) of solitons decreases (increases) with increase in ion temperature.

References

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